No. of Printed Pages : 4

MCS-031

MCA (Revised)

Term-End Examination

December, 2017

04290

MCS-031 : DESIGN AND ANALYSIS OF ALGORITHMS

Time : 3 hours *Maximum Marks : 100*

Note : *Question no. 1 is* ***compulsory****. Attempt any* ***three*** *questions from the rest.*

1. (a) Show that the partition problem is NP. *5*

(b) For the functions $f(x) = 2x^3 + 3x^2 + 1$ and $h(x) = 2x^3 - 3x^2 + 2$, prove that : *2×5=10*

(i) $f(x) = \theta(h(x))$

(ii) $f(x) \neq O(x^2)$

(c) Show that the state entry problem is undecidable. *5*

(d) If S = {a, aa, aaa}, T = {bb, bbb}, then prove that ST = {abb, abbb, aabb, aabbb, aaabb, aaabbb}. *5*

(e) Differentiate between NP-Complete and NP-Hard problems. Give one example for each. *5*

(f) Analyze the time complexity of binary search in worst case. 5

(g) Construct a Deterministic Finite Automata (DFA) over $\Sigma = \{a, b\}$, which accepts all strings over Σ that start and end with the same letter. 5

2. (a) Explain Strassen's Matrix Multiplication Algorithm and apply the same to multiply the following two matrices : 10

$$A = \begin{bmatrix} 5 & 6 \\ -4 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} -7 & 6 \\ 5 & 9 \end{bmatrix}$$

(b) Solve the following 0/1 knapsack problem using dynamic programming : 10

Number of objects n = 6

Weights of objects $w_i = (1,2,5,6,8,10)$

Profit of objects $p_i = (1,6,18,22,30,43)$

Capacity of knapsack, M = 12

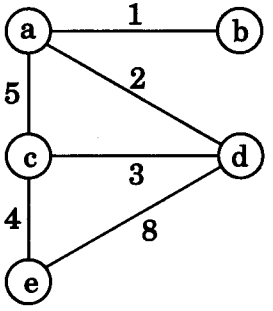
3. (a) Multiply the following two numbers using Karatsuba's method : 5

1026732 and 732912

(b) Define Context-Free Grammar (CFG). Find CFG for the language

$L = \{a^n b^m c^n \mid m, n \geq 1\}$. 5

(c) Write Kruskal's Algorithm and apply the same to find a minimum spanning tree for the following graph : 5



(d) Solve the following recurrence relation : 5

$t_n = 2t_{n-1} + 1$, such that $t_0 = 1$.

4. (a) Write short notes on any ***three*** of the following : *3×5=15*

(i) Chomsky's Classification of Grammar

(ii) Push-Down Automata (PDA)

(iii) Depth-First Search (DFS)

(iv) Asymptotic Notation (O, Ω and θ)

(b) Design a Push Down Automata that accepts the language of odd palindromes. 5

5. (a) Design a Turing Machine that accepts the following language : 8

$L = \{a^n b^n \mid n \geq 1\}$

(b) Write an algorithm for Quick Sort. Sort the following sequence of numbers using Quick Sort :

15, 10, 13, 9, 12, 7

Analyze the time complexity of Quick Sort in best and worst cases. *6+6=12*