No. of Printed Pages : 3

MCS-031

MCA (Revised)

Term-End Examination

June, 2017

03512

MCS-031 : DESIGN AND ANALYSIS OF ALGORITHMS

Time : 3 hours *Maximum Marks : 100*

Note : *Question no. 1 is* ***compulsory****. Attempt any* ***three*** *from the remaining questions.*

1. (a) Use mathematical induction to prove the following expression : 5

$$\sum_{i=1}^{n} 2^i = 2^{n+1} - 1$$

(b) Define Big-O and Big Omega notation, and prove that

$$f(n) = 3 \log n + \log \log n = O(\log n).$$ 6

(c) Write a regular expression to generate strings of odd lengths over the alphabet $\Sigma = \{a, b\}$. 5

(d) Solve the following recurrence equations : 9

(i) $T(n) = 2T(n/2) + n$

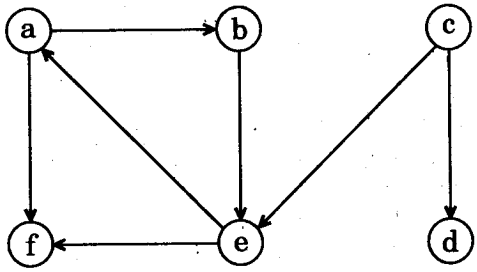
(ii) $T(n) = T(n/2) + 1$

(iii) $T(n) = T(n/2) + n$

(e) Write an algorithm for Merge Sort. Analyze its time complexity. *10*

(f) What is the essence of Greedy technique ? Give an example. *5*

2. (a) Obtain the DFS traversal for the following graph :



Identify the tree edges, back edges and cross edges. *8*

(b) Explain any three applications of DFS traversal. *6*

(c) Explain Kruskal's algorithm to compute the minimum cost spanning tree. *6*

3. (a) Explain how dynamic programming can be used to solve matrix chain multiplication. Apply the algorithm to multiply the following : *10*

3 matrices, $< M_1, M_2, M_3 >$ with

dimensions $< (15, 3), (3, 10), (10, 2) >$

(b) Give a divide and conquer based algorithm to find the i^{th} smallest element in an array of size n. Trace your algorithm to find 3^{rd} smallest in the array

A = {10, 2, 5, 15, 50, 6, 20}. *10*

4. (a) Define Regular Languages. Write regular expressions for the following : *9*

(i) Strings of even length over the alphabet Σ = {a, b}.

(ii) Strings with odd number of a's and even number of b's over the alphabet Σ = {a, b}.

(b) Explain Chomsky's classification for grammars. *6*

(c) Show that the following CFG is ambiguous : *5*

$E \rightarrow E + E$

$E \rightarrow E * E$

$E \rightarrow a/b$

5. (a) Define a Turing machine. *5*

(b) If L_1 and L_2 are context-free languages, then prove that $L_1 \cup L_2$ is also context-free. *5*

(c) Explain the term 'Polynomial time reduction'. Explain how the clique problem can be transformed to the vertex cover problem. *10*